Section 5.2 Addition and Subtraction with Decimals

1. Rule for Adding and Subtracting Decimals: To add (or subtract) decimal numbers, we line up the decimal points and add (or subtract) as usual. The decimal point in the result is written directly below the decimal points in the problem.

Example 1: Simplify each of the following.

a. 3.035+21.9 = 24.935

SDWK

$$\begin{array}{r} 3.035 \\ +\,21.900 \\ \hline 24.935 \end{array}$$

b. 8.4 – 3.71 = 4.69

SDWK

$$\begin{array}{r} {\scriptstyle 7\ 13\ 10} \\ 8.40 \\ -3.71 \\ \hline 4.69 \end{array}$$

2. Applications: Use the rule for adding and subtracting decimals along with the proper operation to solve the given problems.

Example 2: Solve the following applied problem.

A 4-H Club member is raising a lamb to show at the county fair. If she spent \$75 for the lamb, \$25.60 for feed, and \$35.89 for grooming tools, what was the total cost of the project?

Total Cost = \$75.00 + \$25.60 + \$35.89

Total Cost = \$136.49

SDWK

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 75.00 \\ 25.60 \\ +\,35.89 \\ \hline 136.49 \end{array}$$

ANS: The total cost of the project was \$136.49.

Note: Portions of this document are excerpted from the textbook *Prealgebra,* 7th ed. by Charles McKeague

Practice Problems:

Simplify:

a. $-3.21-8.7 = -11.91$

SDWK

$$\begin{array}{r} 3.21 \\ +8.70 \\ \hline 11.91 \end{array}$$

b. $-4.5+2.04 = -2.46$

SDWK

$$\begin{array}{r} {}^{4}\cancel{5}^{10}0 \\ 4.80 \\ -2.04 \\ \hline 2.46 \end{array}$$

Solve the following application problem;

c. A checking account contains $342.38. If checks are written for $25.04, $36.71 and $210, how much money is left in the account?

Account Balance = $\$342.38 + (\$-25.04) + (\$-36.71) + (\$-210)$

Account Balance = $\$342.38 + (\$-271.75)$

Account Balance = $\$70.63$

SDWK

$$\begin{array}{r} {}^{1}25.04 \\ 36.71 \\ +210.00 \\ \hline 271.75 \end{array}$$

$$\begin{array}{r} 2{}^{14}\cancel{5}{}^{13}.13 \\ 342.38 \\ -271.75 \\ \hline 70.63 \end{array}$$

ANS: The account balance would be $70.63.

Answers to Practice Problems:

a. –11.91; b. –2.46; c. $70.63

Note: Portions of this document are excerpted from the textbook *Prealgebra*, 7th ed. by Charles McKeague

Section 5.3 Multiplication with Decimals; Circumference and Area of a Circle

1. **Rule for Multiplying Decimals:** To multiply two decimal numbers:
   - Multiply as you would if the decimal point was not there. Arrange the two numbers one under the other and lined up on the right hand side.
   - Place the decimal point in the answer so that the number of digits to its right is equal to the total number of digits to the right of the decimal points in the original two numbers in the problem. SDWK

Example 1: Simplify each of the following.

a. $7.8 \bullet 3.02 =$ 23.556

$$\begin{array}{r} 3.02 \\ \times\ 7.8 \\ \hline 2416 \\ +21140 \\ \hline 23.556 \end{array}$$ 3 places

b. $-4.5(2.04) =$ −9.18

$$\begin{array}{r} 2.04 \\ \times\ 4.5 \\ \hline 1020 \\ +\ 8160 \\ \hline 9.180 \end{array}$$ 3 places

2. **Estimating:** Round each decimal to a whole number, then perform the computations. Your answer will be an estimate for the true result.

Example 2: Estimate the answer.

a. $7.8 \bullet 3.02 \approx 8 \cdot 3$

≈ 24

b. $8.04 + 17.2 \approx 8 + 17$

≈ 25

Note: Portions of this document are excerpted from the textbook *Prealgebra*, 7th ed. by Charles McKeague

Combined

3. Combined Operations: Use the order of operations agreement and the rules for operations on decimal numbers to simplify expressions involving decimals numbers and addition, subtraction, and multiplication.

Example 3: Simplify.

a. $4.04(0.05-6.6) = 4.04(-6.55)$

$= -26.462$

SDMK

$6.60 - 0.05 = 6.55$

6.55×4.04

$2620 + 0000 + 262000 = 26.4620$

b. $3.6+(2.1)^2 = 3.6 + 4.41$

$= 8.01$

SDMK

$(2.1)^2 = (2.1)(2.1) = 4.41$

$2.1 \times 2.1 = 21 + 420 = 4.41$

$3.60 + 4.41 = 8.01$

4. Applied Problems: Decide whether to add, subtract or multiply to solve the given problem, and then apply the appropriate rule for adding, subtracting, or multiplying decimal numbers. . Show your work in algebraic format: identify the quantity that your variable represents, write an equation, solve your equation, and write your answer in English words.

Example 4: Solve. Show all steps in algebraic format.

If I cup of regular coffee contains 105 milligrams of caffeine how much caffeine is contained in 3.5 cups of coffee?

let x = amount of caffeine contained in 3.5 cups of coffee

$$x = \frac{105 \text{ mg caffeine}}{1 \text{ cup of coffee}} \cdot \frac{3.5 \text{ cups of coffee}}{1}$$

$x = (3.5)\cdot(105)$ mg caffeine

$x = 367.5$ mg caffeine

SDMK

$105 \times 3.5 = 525 + 31\,50 = 367.5$ (1 place)

ANS: In 3.5 cups of coffee, there are 367.5 milligrams of caffeine.

5/34 SDWK 5(6)

2 1 2
+ 3 + 3
4.2849
× 3.14
17 13 96
1 4 28 4 90
+ 12 8 5 4 7 00
13.4 5 4 5 8 6

5. Circumference and Area of a Circle: The circumference of a circle is the distance around the circle. The area of a circle is a measure of the space enclosed by the circle. The diameter of a circle is the distance from one side to the other, through the center. The radius is one-half of the diameter. Formulas for circumference, area and radius are:

$C = 2\pi r$ or $C = \pi d$

$A = \pi r^2$

$r = \frac{1}{2}d$

where A is the area, C is the circumference, r is the radius and d is the diameter.

When solving geometry problems use the following format:

- Write the formula.
- Plug in the known values.
- Solve for the requested quantity. Use the appropriate units with your answer.
- You may use your calculator for these problems. Round your answers to the nearest hundredth. Use the value 3.14 for π.

SDWK 4 places

1
3.14
× 3.12
6 28
3 1 4 0
+ 9 4 2 0 0
9.7 9 6 8

Example 5: Solve the following geometry problem:

a. Find the circumference of a circle of diameter 3.12 feet.

$d = 3.12$ feet

$C = \pi d$

$C \approx (3.14)(3.12 \text{ feet})$

$C \approx 9.7968$ feet

$C \approx 9.80$ feet

or

$C \approx 9.8$ feet

SDWK

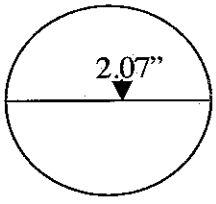
1.035
2 12.070
-2
0
-0
7
-6
10
-10
0

b. Find the area of the given circle.

$d = 2.07$ inches

$r = \frac{1}{2}(2.07 \text{ inches})$

$r = 1.035$ inches



$A = \pi r^2$

$A \approx (3.14)(1.035 \text{ inches})^2$

$A \approx (3.14)(1.071225) \text{ inches}^2$

$A \approx (3.14)(1.071) \text{ inches}^2$

$A \approx 3.36294 \text{ inches}^2$

$A \approx 3.36 \text{ inches}^2$

$(1.035)^2 = 1.071225$

≈ 1.071

Note: Portions of this document are excerpted from the textbook *Prealgebra*, 7th ed. by Charles McKeague

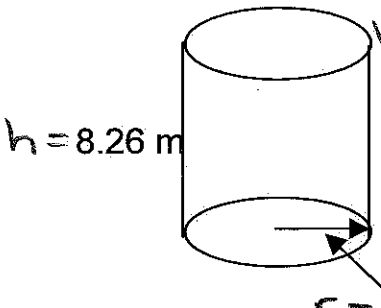
6. Volume of a Right Circular Cylinder: The volume of a right circular cylinder is given by the formula $V = \pi r^2 h$, where r is the radius of the circular top or bottom, and h is the height of the cylinder.

Example 6: Solve the following geometry problems. Use the proper format:

- Write the formula.
- Plug in the known values.
- Solve for the requested quantity. Use the appropriate units with your answer.

You may use your calculator. Round your answers to the nearest hundredth.

a. Find the volume.



$V = \pi r^2 h$

$V \approx (3.14)(2.67\text{m})^2(8.26\text{m})$

$V \approx (3.14)(7.1289)(8.26)\ m^3$

$V \approx (22.384746)(8.26)\ m^3$

$V \approx (22.385)(8.26)\ m^3$

$V \approx 184.900010\ m^3$

$V \approx 184.90\ m^3$

or

$V \approx 184.9\ m^3$

SDWK

1 1
4 4
4 4
267
× 2.67
11869
16020
+53400
7.1289

2 2
3 3
7.1289
× 3.14
285156
1712890
+21386700
22.384746

1 3 6 4
2 5 5
22.385
× 8.26
134131 0
1447700
+179080 00
184.900010

Practice Problems

Simplify:

a. $8.04 \bullet 17.2 = 138.288$

SDWK

8.04
× 17.2
1608
56280
+ 80400
138.288

Note: Portions of this document are excerpted from the textbook *Prealgebra*, 7th ed. by Charles McKeague

SDWK

3.21
×6.2
642
+19260
19.902

b. $-3.21(-6.2) = 19.902$

Estimate:

c. $-3.21 - 3.62 \approx -3 - 4$
≈ -7

d. $-4.5(2.04) \approx -5 \cdot (2)$
≈ -10

SDWK

9.6 − 0.5 = 9.1

9.6 + 0.5 = 10.1

Simplify:

e. $(9.6-0.5)(9.6+0.5)$

$= (9.1)\cdot(10.1)$

$= 91.91$

10.1
× 9.1
101
+9090
91.91

Solve the following applied and geometry problems. Use the proper format:

- Write the formula.
- Plug in the known values.
- Solve for the requested quantity. Use the appropriate units with your answer.

You may use your calculator. Round your answers to the nearest hundredth.

Note: Portions of this document are excerpted from the textbook *Prealgebra*, 7th ed. by Charles McKeague

SDWK

$120 \times 0.08 = 9.60$

$40.00 + 9.60 = 49.60$

f. Suppose it costs \$20 per day and \$0.08 per mile to rent a car. What is the total bill if the car is rented for 2 days and is driven 120 miles?

let X = Total Bill to Rent car

$$X = \left(\frac{\$20}{1\text{ day}}\right)\cdot\left(\frac{2\text{ days}}{1}\right) + \left(\frac{\$0.08}{1\text{ mile}}\right)\cdot\left(\frac{120\text{ miles}}{1}\right)$$

$X = \$40 + \$9.60$

$X = \$49.60$

ANS: The total bill for the car rental is \$49.60.

g. Find the area of a circle of radius 4.07 yards.

$A = \pi r^2$

$A = \pi (4.07\text{ yd})^2$

$A \approx (3.14)(16.5649\text{ yd}^2)$

$A \approx (3.14)(16.565\text{ yd}^2)$

$A \approx 52.0541\text{ yd}^2$

$A \approx 52.01\text{ yd}^2$

SDWK

4.07×4.07 (4 places)

$2849 + 00000 + 162800 = 16.5649$

16.565×3.14 (5 places)

$2\,66\,26\,260 + 1\,65\,650 + 49\,69\,500 = 52.0541$

h. Find the volume of a cylinder with a base of radius 4.32 feet and a height of 7.2 feet.

$V = \pi r^2 h$

$V = \pi (4.32\text{ feet})^2 (7.2\text{ feet})$

$V = \pi (18.6624\text{ ft}^2)(7.2\text{ ft})$

$V \approx (3.14)(18.662)(7.2)\text{ ft}^3$

$V \approx (3.14)(134.3664)\text{ ft}^3$

$V \approx (3.14)(134.366)\text{ ft}^3$

$V \approx 421.90924\text{ ft}^3$

$V \approx 421.91\text{ ft}^3$

SDWK

4.32×4.32 (4 places)

$2884 + 12960 + 172800 = 18.6624$

18.662×7.2 (4 places)

$37324 + 1306340 = 134.3664$

134.366×3.14 (5 places)

$1537464 + 1343660 + 40309800 = 421.90924$

Answers to Practice Problems:
a. 138.288; b. 19.902; c. –7; d. –10; e. 91.91; f. \$49.60; g. 52.01 yd^2 h. 421.92 ft^3

Note: Portions of this document are excerpted from the textbook *Prealgebra*, 7th ed. by Charles McKeague

Section 5.4 Division with Decimals

1. Dividing by a Whole Number: To divide a decimal number by a whole number

- Divide as you would if the decimal point was not there. If the decimal number has digits after the decimal place, you may write as many zeroes as you need on the right side of the decimal number.
- Divide until there is no remainder or until the answer begins a repeating sequence unless the instructions say to round to a certain decimal place.
- The decimal point in the answer is placed directly above the decimal point in the dividend.

Example 1: Simplify each of the following.

a. $190.8 \div 45 = 4.24$

SDWK

$$45\overline{)190.80}\quad 4.24$$
$$-180$$
$$108$$
$$90$$
$$180$$
$$-180$$
$$0$$

b. $16 \div 6 = 2.\overline{6}$

$$6\overline{)16.0}\quad 2.\overline{6}$$
$$-12$$
$$40$$
$$-36$$
$$4$$

Repeat

Example 2: Simplify each of the following. Round your answer to the nearest hundredth.

a. $0.543 \div 21 \approx 0.025\ldots$
≈ 0.03

SDWK

$$21\overline{)10.543}\quad 0.025\ldots$$
$$-42$$
$$123$$
$$-105$$
$$18$$

b. $16 \div 6 \approx 2.666\ldots$
≈ 2.67

SDWK

$$6\overline{)16.000}\quad 2.666\ldots$$
$$-12$$
$$40$$
$$-36$$
$$40$$
$$-36$$
$$40$$
$$-36$$
$$4$$

Note: Portions of this document are excerpted from the textbook *Prealgebra*, 7th ed. by Charles McKeague

2. Dividing by a Decimal: If the divisor is a decimal, change it to a whole number by moving the decimal point to the right as many places as necessary. Then, also move the decimal point in the dividend the same number of places in the same direction. Next, complete the division process using the rule for dividing by a whole number.

Why does this process work? It works because moving the decimal point to the right is the same as multiplying by a power of ten, i.e. moving the decimal one place to the right is the same as multiplying by ten, moving two places is the same is multiplying by 100, and so on. If you move the decimal point the same number of places to the right in both the dividend and the divisor, that's like multiplying the numerator and the denominator of a fraction by the same power of ten.

Example 3: Use your calculator to compute each of the following and compare your answers.

a. $0.3778 \div .25 = 1.5112$

b. $3.778 \div 2.5 = 1.5112$

c. $37.78 \div 25 = 1.5112$

d. $377.8 \div 250 = 1.5112$

SAME

$$0.25\overline{)0.3778}$$

$$25\overline{)37.7800}$$

$$\begin{array}{r} 1.5112 \\ -25 \\ \hline 127 \\ -125 \\ \hline 28 \\ -25 \\ \hline 30 \\ -25 \\ \hline 50 \\ -50 \\ \hline 0 \end{array}$$

Note: Portions of this document are excerpted from the textbook *Prealgebra,* 7th ed. by Charles McKeague

Example 4: Divide. Remember to make the divisor a whole number before you perform the division.

$0.3778 \div .25 = 1.5112$

SDWK

$$
\begin{array}{r}
1.5112 \\
25 \overline{)\,37.7800} \\
-25 \\
\hline
127 \\
-125 \\
\hline
28 \\
-25 \\
\hline
30 \\
-25 \\
\hline
50 \\
-50 \\
\hline
0
\end{array}
$$

Example 5: Divide. Round your answer to the nearest hundredth, if necessary.

$0.49 \div 0.048 \approx 10.208$

≈ 10.21

SDWK

$0.048 \overline{)\,0.490}$

$$
\begin{array}{r}
10.208\ldots \\
48 \overline{)\,490.000} \\
-48 \\
\hline
10 \\
-0 \\
\hline
100 \\
-96 \\
\hline
40 \\
-0 \\
\hline
400 \\
-384 \\
\hline
16
\end{array}
$$

3. Calculating Grade Point Averages: Set up a chart showing, for each class, the number of units, the final grade, and the grade points. Calculate grade points by multiplying the number of units and the point value for the grade earned. The point values are:

A	**4 points**
B	**3 points**
C	**2 points**
D	**1 point**
F	**0 points**

Add the grade points earned for each course to get a total grade point. Divide this total by the sum of the units to get the grade point average.

Note: Portions of this document are excerpted from the textbook *Prealgebra,* 7th ed. by Charles McKeague

Example 6: Calculate the grade point average. Use your calculator. Round your answer to the nearest hundredth.

Algebra	5 units	B ← 3 pts
Chemistry	4 units	C ← 2 pts
English	3 units	A ← 4 pts
History	3 units	B ← 3 pts

$$GPA = \frac{(5\text{ units})(B) + (4\text{ units})(C) + (3\text{ units})(A) + (3\text{ units})(B)}{15\text{ units}}$$

$$GPA = \frac{5(3\text{ pts}) + 4(2\text{ pts}) + 3(4\text{ pts}) + 3(3\text{ pts})}{15}$$

$$GPA = \frac{(15\text{ pts}) + (8\text{ pts}) + (12\text{ pts}) + (9\text{ pts})}{15}$$

$$GPA = \frac{44\text{ pts}}{15}$$

$$GPA \approx 2.933$$

$$GPA \approx 2.93$$

SDWK

$$15\overline{)44.000} = 2.933\ldots$$

-30

140

-135

50

-45

50

-45

5

4. Applied Problems: Decide whether to add, subtract or multiply to solve the given problem, and then apply the appropriate rule for adding, subtracting, or multiplying decimal numbers. When solving applied problems, use algebraic format: identify the quantity that your variable represents, write an equation, solve it, and write your answer in English words. Remember that the steps count for points!!

Example 7: If gas costs $2.29 per gallon, how much does 18.9 gallons cost? Round to the nearest cent, if necessary.

Let x = cost of 18.9 gallons of gas

$$x = \left(\frac{\$2.29}{1\text{ gallon}}\right)\left(\frac{18.9\text{ gallons}}{1}\right)$$

$$x = \$43.281$$

$$x \approx \$43.28$$

ANS: It cost $43.28 for 18.9 gallons of gas.

SDWK

$$\begin{array}{r} 2\ 7 \\ 2\ 8 \\ 2.29 \\ \times\ 18.9 \\ \hline 12061 \\ 18320 \\ +\ 22900 \\ \hline 43.281 \end{array}$$

3 places

Note: Portions of this document are excerpted from the textbook *Prealgebra*, 7th ed. by Charles McKeague

Practice Problems:

a. Divide: $146.38 \div 26$

$= 5.63$

$$
\begin{array}{r}
5.63 \\
26 \overline{)\,146.38} \\
-130 \\
\hline
163 \\
-156 \\
\hline
78 \\
-78 \\
\hline
0
\end{array}
$$

b. Divide: $\dfrac{2.40}{0.74}$

$= 3.243$

$$
\begin{array}{r}
3.243 \\
74 \overline{)\,240.000} \\
-222 \\
\hline
180 \\
-148 \\
\hline
320 \\
-296 \\
\hline
240 \\
-222 \\
\hline
18
\end{array}
$$

Repeats

c. Solve the applied problem using algebraic format: How many hours does a person making $6.78 per hour have to work in order to earn $257.64?

let x = hours a person works to earn $257.64

$\$257.64 = \$6.78x$

$\dfrac{\$257.64}{\$6.78} = \dfrac{\$6.78x}{\$6.78}$

$38 = x$

check

$\$257.64 = \$6.78(38)$

$\$257.64 = \$257.64$

TRUE!

$/hr

$$
\begin{array}{r}
38.0 \\
678 \overline{)\,25764.0} \\
-2034 \\
\hline
5424 \\
-5424 \\
\hline
0
\end{array}
$$

$$
\begin{array}{r}
678 \\
\times\ 38 \quad \text{2 places} \\
\hline
5424 \\
+20340 \\
\hline
25764
\end{array}
$$

ANS: To make $257.64, a person earning $6.78 per hour needs to work 38 hours.

Note: Portions of this document are excerpted from the textbook *Prealgebra*, 7th ed. by Charles McKeague

d. Divide and round to the nearest hundredth: $\frac{7.26}{2.3} \approx 3.156\ldots$

≈ 3.16

$2.3\overline{)7.26}$

$23\overline{)72.600}$

3.156

-69

36

-23

130

-115

150

-138

12

Answers to Practice Problems:

a. 5.63; b. $3.\overline{243}$ (Don't round. Show the repeating portion with a bar.)
c. 38 hours; d. 3.16 (after rounding)

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Section 5.5 Fractions, Decimals, and the Volume of a Sphere

1. Converting Fractions to Decimals: To convert a fraction to a decimal, divide the numerator by the denominator. You must divide until the decimal terminates or repeats unless the instructions ask you to round to a given decimal place.

SDWK

Example 1: Convert the given fraction to a decimal.

a. $\frac{7}{9} = 0.\overline{7}$

$$\begin{array}{r} 0.7 \\ 9\overline{)\,7.0} \\ -63 \\ \hline 7 \end{array}$$ Repeats

b. $\frac{5}{12} = 0.41\overline{6}$

$$\begin{array}{r} 0.416 \\ 12\overline{)\,5.000} \\ -48 \\ \hline 20 \\ -12 \\ \hline 80 \\ -72 \\ \hline 8 \end{array}$$ Repeats

Example 2: Convert the given fraction to a decimal. Round to the nearest thousandth, if necessary.

SDWK

a. $\frac{5}{7} \approx 0.71\underset{\downarrow}{4}2\ldots$ (5)
≈ 0.714

$$\begin{array}{r} 0.7142\ldots \\ 7\overline{)\,5.0000} \\ -49 \\ \hline 10 \\ -7 \\ \hline 30 \\ -28 \\ \hline 20 \\ -14 \\ \hline 6 \end{array}$$

b. $\frac{7}{64} \approx 0.109\underset{\downarrow}{3}\ldots$ (7)
≈ 0.109

$$\begin{array}{r} 0.1093\ldots \\ 64\overline{)\,7.0000} \\ -64 \\ \hline 60 \\ -0 \\ \hline 600 \\ -576 \\ \hline 240 \\ -192 \\ \hline 48 \end{array}$$

2. Converting Decimals to Fractions: To convert a decimal to a fraction,

- Numerator of the fraction: Place the digits to the right of the decimal point.
- Denominator of the fraction: Write the place value named by the last digit in the decimal if the "ths' is left off.
- Reduce the fraction to lowest terms.

SDWK

Example 3: Convert to a fraction in lowest terms.

a. $3.045 = 3\frac{45}{1000}$
$= \frac{3045}{1,000}$
$= \frac{3 \cdot 5 \cdot 7 \cdot 29}{2 \cdot 5 \cdot 2 \cdot 5 \cdot 2 \cdot 5}$
$= \frac{609}{200}$

3,045
5 609
3 203
7 29

1000
10 100
2 5 10 10
2 5 2 5

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2,125 → 5 · 425 → 5 · 85 → 5 · 17

1,000 → 10 · 100 → 2 · 5 · 10 · 10 → 2 · 5 · 25 · 25

b. $2.125 = 2\frac{125}{1{,}000}$

$= \frac{2{,}125}{1{,}000}$

$= \frac{5 \cdot 5 \cdot 5 \cdot 17}{2 \cdot 5 \cdot 2 \cdot 5 \cdot 2 \cdot 5}$ $= \frac{17}{2 \cdot 2 \cdot 2}$ $= \frac{17}{8}$

3. Problems Containing Both Fractions and Decimals: To work problems that have both fractions and decimals, you may

- Change all of the decimals to fractions and simplify.
- Change the fractions to decimals, **if they make terminating decimals,** and then simplify. If the fractions do not make terminating decimals, don't use this technique.
- Try to divide out any common factors, and then simplify.

Common factors can be divided out even when one or both of the numbers are decimals.

Example 4: $\frac{1}{5} \bullet (3.5) = \frac{1}{5} \bullet \frac{3.5}{1}$

$= \frac{1}{5 \div 5} \bullet \frac{3.5 \div 5}{1}$

$= \frac{1}{1} \bullet \frac{0.7}{1}$

$= 0.7$

or $\frac{1}{5} \cdot (3.5)$ $= \frac{1}{5} \cdot \left(\frac{35}{10}\right)$ $= \frac{5 \cdot 7}{5 \cdot 2 \cdot 5}$ $= \frac{7}{10} = 0.7$

SPWL

$3.5 = 3\frac{5}{10}$

$= \frac{35}{10}$

10 → 2 · 5 | 35 → 5 · 7

Example 5: Simplify by dividing out common factors. Give an exact answer; that is, do not round.

SPWL

a. $\frac{1}{6} \bullet 1.8 = \frac{1}{6} \cdot \left(\frac{18}{10}\right)$

$= \frac{1 \cdot 2 \cdot 3 \cdot 3}{2 \cdot 3 \cdot 2 \cdot 5}$

$= \frac{3}{10}$

$= 0.3$

$1.8 = 1\frac{8}{10}$

$= \frac{18}{10}$

6 → 2 · 3 | 18 → 2 · 9 → 3 · 3 | 10 → 2 · 5

b. $\frac{2}{3} \bullet 2.1 = \frac{2}{3} \cdot \left(\frac{21}{10}\right)$

$= \frac{2 \cdot 3 \cdot 7}{3 \cdot 2 \cdot 5}$

$= \frac{7}{5}$ $= 1.4$

$2.1 = 2\frac{1}{10}$

$= \frac{21}{10}$

21 → 3 · 7 | 10 → 2 · 5

1.4
5 | 7.0
−5
20
−20
0

Note: Portions of this document are excerpted from the textbook *Prealgebra*, 7th ed. by Charles McKeague

c. $\left(\frac{4}{9}\right)(1.8)+\frac{5}{6}(2.4)$

$= \frac{4}{9}\cdot\left(\frac{18}{10}\right) + \frac{5}{6}\left(\frac{24}{10}\right)$

$= \frac{2\cdot 2\cdot 2\cdot 3\cdot 3}{3\cdot 3\cdot 2\cdot 5} + \frac{5\cdot 2\cdot 2\cdot 2\cdot 3}{2\cdot 3\cdot 2\cdot 5}$

$= \frac{4}{5} + \frac{10}{5}$

$= \frac{4+10}{5}$

$= \frac{14}{5}$

$= 2.8$

$1.8 = 1\frac{8}{10} = \frac{18}{10}$

$2.4 = 2\frac{4}{10} = \frac{24}{10}$

2.8
$5\overline{)14.0}$
-10
40
-40
0

Sometimes, there are no common factors to divide out or it is simply easier to convert the fractions to decimals and complete the arithmetic. As long as the fractions convert to terminating decimals, this technique will work well. However, if the problem contains fractions that convert to repeating decimals, then you will introduce error in the answer when you round that decimal in order to do the remaining calculations. These problems are best done by calculator where the calculator can carry 15 or more decimal places, minimizing the error in the answer.

Try these problems. If the problem contains a fraction that converts to a repeating decimal, use a calculator and round your answer to the nearest hundredth.

Example 6: Simplify.

$\frac{19}{20}(1.32+0.48)$

$= \frac{19}{20}(1.8)$

$= \frac{19}{20}\cdot\left(\frac{18}{10}\right)$

$= \frac{19\cdot 18\div 2}{20\cdot 10\div 2}$

$= \frac{19\cdot 9}{20\cdot 5}$

$= \frac{171}{100}$

$= 1.71$

$1.32 + 0.48 = 1.80$

$1.8 = 1\frac{8}{10} = \frac{18}{10}$

Example 7: Simplify. Round your answer to the nearest hundredth.

$\frac{2}{3}(1.4)+\frac{1}{2}(0.5)$

$= \frac{2.8}{3} + \frac{0.5}{2}$

$\approx 0.933 + 0.25$

≈ 1.183

≈ 1.18

0.933
$+0.250$
1.183

$1.4 \times 2 = 2.8$

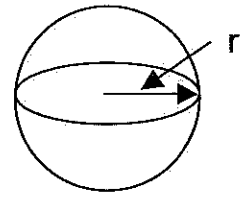
0.25
$2\overline{)0.50}$
-4
10
-10
0

0.933
$3\overline{)2.80}$
-27
10
-9
10
-9
10
-9
1

Note: Portions of this document are excerpted from the textbook *Prealgebra*, 7th ed. by Charles McKeague

4. **Volume of a sphere:** The volume of a sphere of radius r is given by

$$V = \frac{4}{3}\pi r^3$$



To solve these geometry problems, remember to write out the following steps:

- Write the formula,
- Plug in the known values using 3.14 for π, and
- Use your calculator to complete the calculations and write your answer with the correct units. Round to the nearest hundredth.

Example 8: Find the volume of a sphere of radius 17.859 feet

$V = \frac{4}{3}\pi r^3$, $r = 17.859$ ft

$V \approx \frac{4}{3}(3.14)(17.859\text{ ft})^3$

$V \approx \frac{4}{3}(3.14)(5,696.0187707\ldots)\text{ ft}^3$

$V \approx \frac{4}{3}(17,885.49894)\text{ ft}^3$

$V \approx \frac{71,541.99576}{3}\text{ ft}^3$

$V \approx 23,847.33192\text{ ft}^3$

$V \approx 23,847.33\text{ ft}^3$

Practice Problems:

a. Convert the given fraction to a decimal. Do not round.

$\frac{5}{18} = 0.2\overline{7}$

$$\begin{array}{r} 0.27 \\ 18\overline{)5.000} \\ -36 \\ \hline 140 \\ -126 \\ \hline 140 \end{array}$$

Repeats

SDWK

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b. Convert the given fraction to a decimal. Round to the nearest hundredth. SDWK

$\frac{5}{17} \approx 0.2\underset{\curvearrowright}{9}4\ldots$

≈ 0.29

$$\begin{array}{r} 0.29\underset{\curvearrowright}{4} \\ 17\overline{)5.000} \\ -34 \\ \hline 160 \\ -153 \\ \hline 70 \\ -68 \\ \hline 2 \end{array}$$

c. Simplify. Give an exact answer. $\frac{1}{7} \cdot 1.4$

$\frac{1}{7} \cdot 1.4 = \frac{1}{7} \cdot \left(1\frac{4}{10}\right)$

$= \frac{1}{7}\left(\frac{14}{10}\right)$

$= \frac{\cancel{14}^{2} \cdot \cancel{7}}{\cancel{7} \cdot 2 \cdot 5}$

$= \frac{1}{5}$

$= 0.2$

$1\frac{4}{10} = \frac{1 \cdot 10 + 4}{10} = \frac{14}{10}$

$14 = 2 \cdot 7$

$10 = 2 \cdot 5$

$$\begin{array}{r} 0.2 \\ 5\overline{)1.0} \\ -10 \\ \hline 0 \end{array}$$

14 → 2 7; 10 → 2 5

d. Simplify. Give an exact answer. $\frac{1}{2} + (0.75)\left(\frac{2}{5}\right)$

$\frac{1}{2} + (0.75)\left(\frac{2}{5}\right)$

$= \frac{1}{2} + \left(\frac{75}{100}\right)\left(\frac{2}{5}\right)$

$= \frac{1}{2} + \frac{3 \cdot \cancel{5} \cdot \cancel{5} \cdot 2}{2 \cdot 5 \cdot \cancel{2} \cdot \cancel{5} \cdot 5}$

$= \frac{1}{2} + \frac{3}{2 \cdot 5}$

$= \frac{1}{2} \cdot \frac{5}{5} + \frac{3}{10}$

$= \frac{5}{10} + \frac{3}{10}$

$= \frac{8}{10}$

$= 0.8$

$75 = 3 \cdot 5 \cdot 5$

$100 = 2 \cdot 5 \cdot 2 \cdot 5$

75 → 3 25 → 5 5; 100 → 10 10 → 25 25

e. Simplify. Use a calculator and round your answer to the nearest hundredth. $\frac{3}{7}(4.1 - 3.3) + \frac{1}{2}(2.4)$

$= \frac{3}{7}(0.8) + \frac{2.4}{2}$

$= \frac{3}{7}(0.8) + 1.2$

$= \frac{2.4}{7} + 1.2$

$\approx 0.3\underset{\curvearrowright}{4}28\ldots + 1.2$

$\approx 0.34 + 1.2$

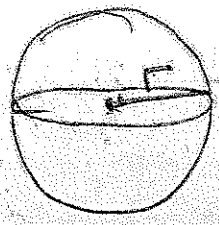
≈ 1.54

$$\begin{array}{r} 0.34 \\ +1.20 \\ \hline 1.54 \end{array}$$

$$\begin{array}{r} {\scriptstyle 3\ 11} \\ 4.1 \\ -3.3 \\ \hline 0.8 \end{array} \qquad \begin{array}{r} 0.8 \\ \times 3 \\ \hline 2.4 \end{array}$$

$$\begin{array}{r} 1.2 \\ 2\overline{)2.4} \\ -2 \\ \hline 4 \\ -4 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 0.3428\ldots \\ 7\overline{)2.4000} \\ 21 \\ \hline 30 \\ -28 \\ \hline 20 \\ -14 \\ \hline 60 \\ -56 \\ \hline 4 \end{array}$$



$r = \frac{1}{2}d$

$r = \frac{1}{2}(23.45\text{yds})$

$r = 11.725\text{yds}$

$d = 23.45\text{ yds}$

sdwk

$$2\overline{)23.450}\quad 11.725$$

-2, 3, -2, 14, -14, 5, -4, 10, -10, 0

Round to nearest Hundredth

f. Find the volume of a sphere of diameter 23.45 yds.
(Hint: First find the radius of the sphere.)

$V = \frac{4}{3}\pi r^3$, $r = 11.725\text{yds}$

$V = \frac{4}{3}(3.14)(11.725\text{yds})^3$

$V \approx \frac{4}{3}(3.14)(1{,}611.901703)\,yd^3$

$V \approx \frac{4}{3}(5{,}061.371347)\,yd^3$

$V \approx 20{,}245.485388\,yd^3$

$V \approx 6{,}748.4^{3}9^{4}51\,yd^3$

$V \approx 6{,}748.50\,yd^3$

or

$V \approx 6{,}748.5\,yd^3$

Answers to Practice Problems: a. $0.2\overline{7}$; b. 0.29; c. 0.2; d. 0.8; e. 1.54; f. 6748.50 yds^3

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Section 5.6 Equations Containing Decimals

1. Solving Equations That Contain Decimals: Use the rules for solving equations along with the rules for adding, subtracting, multiplying and dividing decimals to solve equations that contain decimals.

SPWK

$7.1x - 5.2$

1.9

Example 1: Solve each of the following.

a. $x+7.1=5.2$

$-7.1 + x + 7.1 = -7.1 + 5.2$

$x = -1.9$

check

$(-1.9)+7.1=5.2$

$5.2=5.2$

TRUE!

ANS: The solution is -1.9.

b. $.3y=.273$

$\frac{0.3y}{0.3} = \frac{0.273}{0.3}$

$y = 0.91$

SPWK

$0.3\overline{)0.273}$

$3\overline{)2.73}$

0.91

-27

3

-3

0

check

$0.3(0.91)=0.273$

$0.273=0.273$

TRUE!

ANS: The solution is 0.91.

c. $6n+0.88=2n-0.77$

$-2n + 6n + 0.88 = -2n + 2n - 0.77$

$4n + 0.88 = -0.77$

$-0.88 + 4n + 0.88 = -0.88 + (-0.77)$

$4n = -1.65$

$\frac{4n}{4} = \frac{-1.65}{4}$

$n = -0.4125$

check

$6(-.4125)+0.88=2(-.4125)-0.77$

$-2.475+0.88=-0.825-0.77$

$-1.595=-1.595$

TRUE!

ANS: The solution is -0.4125.

SPWK

0.88

0.77

1.65

$4\overline{)1.6500}$

0.4125

-16

5

-4

10

-8

20

-20

0

2. Applied Problems: The two types of applied problems covered in this section include the service charge type and the coin type. You may use your calculator to do the calculations on these applied problems.

Note: Portions of this document are excerpted from the textbook *Prealgebra*, 7th ed. by Charles McKeague

Service Charge Problems: The formula for the cost of service is given by:

Total service charge = (per visit charge)(no. of visits) + (per hour charge)(no. of hours)

When solving these problems, each of the steps below is worth points, so be sure to show all of the steps.

- Write a statement telling what quantity your variable(s) represent.
- Write an equation that describes the situation given in the problem. Plug in any known values.
- Solve the equation, showing steps.
- Write your solution in English words.

Example 2: A cable company charges $32.50 for a service charge, then $35.25 for each hour it takes their technician to make the repair. If your total bill is $173.50, how many hours did it take the technician to fix your cable?

Let x = hours it takes the technician to fix the cable

Total service charge = $173.50
per visit charge = $32.50
per hour charge = $35.25

$173.50 = ($32.50)(1) + ($35.25)(x)

$173.50 = 32.50 + 35.25x

$-32.50 + 173.50 = -32.50 + 32.50 + 35.25x$

$141 = 35.25x$

$\frac{141}{35.25} = \frac{35.25x}{35.25}$

$4 = x$

Ans: It took the technician 4 hours to fix the cable.

SDWK

173.50
− 32.50
141.00

35.25 | 141.00

3,525 | 14100.0
−14100
0

check

$173.50 = $32.50 + $35.25(4)

$173.50 = $32.50 + $141.00

$173.50 = $173.50

TRUE!

212
35.25 (2 places)
× 4
141.00

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Coin Problems: To solve a coin problem, set up a chart listing the types of coins in the rows and the number of coins, the value of one coin, and the total value of each type of coin in the columns.

- Identify the quantity your variable(s) represents by putting the variable in the box that describes the quantity.
- Fill in all of the other boxes using known information and your variable.
- Write an equation that describes the situation given in the problem.
- Solve the equation, showing steps.
- Write your solution in English words

Example 3: A collection of dimes and quarters has a total value of $95.20. If there are three times as many quarters as dimes, how many of each coin is in the collection?

Coins	Value of one coin	Number of coins	Value of collection
Dimes	$0.10	x	$0.10x$
Quarters	$0.25	$3x$	$0.25(3x)$
Total	— NA —	— NA —	$95.20

Let x = number of dimes in the collection

$95.20 = 0.10x + 0.25(3x)$

$95.20 = 0.10x + 0.75x$

$95.20 = 0.85x$

$\frac{95.20}{0.85} = \frac{0.85x}{0.85}$

$112 = x$

$3x = 3(112)$

$= 336$

SDWK

$0.25 \times 3 = 0.75$

$0.10 + 0.75 = 0.85$

$0.85\overline{)95.20}$

$85\overline{)9520}$ → 112

-85

102

-85

170

-170

0

check:

$\$95.20 = \$0.10(112) + \$0.25[3(112)]$

$\$95.20 = \$11.20 + \$0.25(336)$

$\$95.20 = \$11.20 + \$84.00$

$\$95.20 = \$95.20$

TRUE!

ANS: There are 112 dimes and 336 quarters in the coin collection.

112×0.10 (2 places) $= 11.20$

$84.00 + 11.20 = 85.20$

336×0.25 (2 places) $= 84.00$

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Practice Problems:

a. Solve: $\frac{1}{2}x - 3.78 = 2.52$

$\frac{1}{2}x - 3.78 + 3.78 = 2.52 + 3.78$

$\frac{1}{2}x = 6.3$

$\frac{2}{1} \cdot \frac{1}{2}x = 2 \cdot (6.3)$

$x = 12.6$

ANS! The solution is 12.6.

Check

$\frac{1}{2}(12.6) - 3.78 = 2.52$

$\frac{12.6}{2} - 3.78 = 2.52$

$6.3 - 3.78 = 2.52$

$2.52 = 2.52$

TRUE!

SDWK

$\begin{array}{r} {\scriptstyle 1\ 1} \\ 3.78 \\ +\ 2.52 \\ \hline 6.30 \end{array}$

$\begin{array}{r} 6.3 \\ \times\ 2 \\ \hline 12.6 \end{array}$

$2\overline{)12.6}\ 6.3$; -12 ; $6 - 6 = 0$

$\begin{array}{r} 5\ 12\ 10 \\ 6.30 \\ -3.78 \\ \hline 2.52 \end{array}$

b. A car rental company charges \$52 per day and \$0.43 per mile for a rental car. If the rental charge was \$389.03 for a four-day rental, how many miles was the car driven?

Let x = miles the rental car was driven over four days

$\$389.03 = \$52 \cdot (4) + \$0.43 \cdot (x)$

$389.03 = 208 + 0.43x$

$-208 + 389.03 = -208 + 208 + 0.43x$

$181.03 = 0.43x$

$\frac{181.03}{0.43} = \frac{0.43x}{0.43}$

$421 = x$

Check:

$\$389.03 = \$52(4) + \$0.43(421)$

$\$389.03 = \$208 + \$181.03$

$\$389.03 = \$389.03$

TRUE!

SDWK

$\begin{array}{r} 5\ 2 \\ \times\ 4 \\ \hline 208 \end{array}$

$\begin{array}{r} 389.03 \\ -208.00 \\ \hline 181.03 \end{array}$

$0.43\overline{)181.03}$; $43\overline{)18103}\ 421$; -172 ; $90 - 86 = 4$; $43 - 43 = 0$

$\begin{array}{r} 421 \\ +\ 0.43 \\ \hline 1263 \\ 16840 \\ \hline 181.03 \end{array}$

ANS! The rental car was driven 421 miles over four days.

c. A collection of nickels and quarters has a total value of $17.80. If there are ten more quarters than nickels, how many of each coin is in the collection?

Let x = number of nickels in the collection

Coins	Value of one coin	number of coins	Value of the collection
nickels	$0.05	x	$0.05x
quarters	$0.25	$x+10$	$0.25(x+10)
total	-NA-	-NA-	$17.80

$$\$17.80 = \$0.05x + \$0.25(x+10)$$

$$17.80 = 0.05x + 0.25x + 2.5$$

$$17.8 = 2.5 + 0.3x$$

$$-2.5 + 17.8 = -2.5 + 2.5 + 0.3x$$

$$15.3 = 0.3x$$

$$\frac{15.3}{0.3} = \frac{0.3x}{0.3}$$

$$51 = x$$

$$x + 10 = (51) + 10 = 61$$

check

$\$17.80 = \$0.05(51) + \$0.25[(51)+10]$

$\$17.80 = \$2.55 + \$0.25[61]$

$\$17.80 = \$2.55 + \$15.25$

$\$17.80 = \$17.80$

TRUE!

ANS: There are 51 nickels and 61 quarters in the collection.

SDWK

0.25 ×10 → 2 places; 00 + 250 = 2.50

0.05 + 0.25 = 0.30

17.80 − 2.50 = 15.30

0.3 | 15.3

51
3 | 153
−15
3
−3
0

0.05 ×51 → 2 places; 005 + 250 = 2.55

0.25 ×61 → 2 places; 25 + 1500 = 15.25

Answers to Practice Problems:

a. {12.6 ⅔}; b. 421 miles; c. There are 61 quarters and 51 nickels.

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Section 5.7 Square Roots and the Pythagorean Theorem

1. Definition of Square Root: The square root of a positive number a, written $\sqrt{a}$, is the number we square to get a. In symbols:

If $\sqrt{a} = b$ then $b^2 = a$.

A table of common square roots is given below.

Table of Common Square Roots

Statement	In Words	Reason
$\sqrt{0} = 0$	The square root of 0 is 0	Because $0^2 = 0$
$\sqrt{1} = 1$	The square root of 1 is 1	Because $1^2 = 1$
$\sqrt{4} = 2$	The square root of 4 is 2	Because $2^2 = 4$
$\sqrt{9} = 3$	The square root of 9 is 3	Because $3^2 = 9$
$\sqrt{16} = 4$	The square root of 16 is 4	Because $4^2 = 16$
$\sqrt{25} = ?$	The square root of 25 is ___	Because $?^2 = 25$
$\sqrt{36} = ?$	The square root of 36 is ___	Because $?^2 = 36$

Example 1: Simplify each of the following.

a. $\sqrt{49} = 7$, since $7^2 = 49$

b. $\sqrt{81} + \sqrt{16} = 9 + 4 = 13$, since $9^2 = 81$ & $4^2 = 16$

c. $\sqrt{9+16} = \sqrt{25} = 5$, since $5^2 = 25$

d. $\sqrt{\frac{16}{25}} = \frac{4}{5}$, since $\frac{4^2}{5^2} = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$

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2. Perfect Squares: A number whose square root is a whole number is called a perfect square. The perfect squares are:
1, 4, 9, 16, 25, 36, 49, 64, 81, 100, ... Can you name the next two perfect squares?

3. Square Roots of Numbers That Are Not Perfect Squares:
If a number is not a perfect square, then its square root is a non-repeating, nonterminating decimal. Use your calculator to get a decimal approximation for the square root. Round your answer to the decimal place indicated in the directions.

Example 2: Use a calculator to approximate each of the following to three decimal places (nearest thousandth).

a. $\sqrt{37} \approx 6.0827\ldots$
≈ 6.083

b. $\sqrt{3}+\sqrt{26} \approx 1.7320\underline{5}\ldots + 5.0990\underline{1}\ldots$
$\approx 1.7321 + 5.0990$
≈ 6.8311
≈ 6.831

c. $\sqrt{15+6}$
$\approx \sqrt{21}$
$\approx 4.5825\ldots$ ≈ 4.583

d. $\sqrt{\frac{5}{8}} = \sqrt{0.625} \approx 0.7905\ldots \approx 0.791$

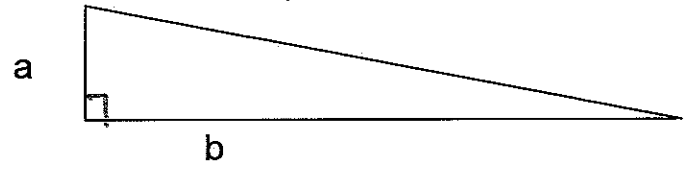
SD WK

$$\begin{array}{r} 1.7321 \\ +\ 5.0990 \\ \hline 6.8311 \end{array}$$

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ -48 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

4. The Pythagorean Theorem: A right triangle is a triangle that contains a 90° angle. The longest side, the one that is opposite the 90° angle, is the hypotenuse; and we use the letter c to denote it. The two shorter sides, the ones that form the 90° angle, are called the legs; and we use the letters a and b to denote them. The Pythagorean Theorem states that the hypotenuse is the square root of the sum of the squares of the legs. In symbols:

$$c = \sqrt{a^2 + b^2}$$



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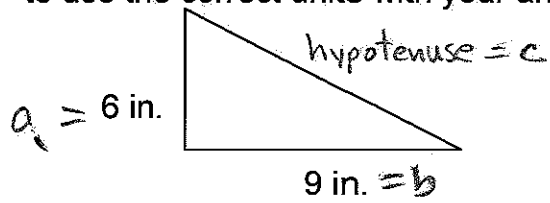
Example 3: Find the length of the hypotenuse in the given triangle. Use the proper format: write the formula, plug in the known values, find the hypotenuse. You may use your calculator to perform the calculations. Round your answer to the nearest hundredth. Be sure to use the correct units with your answer.



$c = \sqrt{a^2 + b^2}$

$c = \sqrt{(6\text{ in})^2 + (9\text{ in})^2}$

$c = \sqrt{36\text{ in}^2 + 81\text{ in}^2}$

$c = \sqrt{117\text{ in}^2}$

$c = \sqrt{117}\text{ in}$

$c \approx 10.816\ldots\text{ in}$

$c \approx 10.82\text{ in}$

ANS: The hypotenuse is approximately 10.82 inches.

Practice Problems:

Simplify. Give exact answers.

a. $\sqrt{64} = 8$, since $8^2 = 64$

b. $\sqrt{25} + \sqrt{4} = 5 + 2 = 7$, since $5^2 = 25$ & $2^2 = 4$

c. $\sqrt{64+36} = \sqrt{100} = 10$, since $10^2 = 100$

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d. $\sqrt{\frac{25}{49}} = \frac{5}{7}$, since $\frac{5^2}{7^2} = \frac{25}{49}$

Simplfy using a calculator. Round your answer to three decimal places.

e. $\sqrt{125} \approx 11.1803\ldots$
≈ 11.180, or
≈ 11.18

f. $\sqrt{7}+\sqrt{52} \approx 2.6457\ldots + 7.2111\ldots$
≈ 9.8568
≈ 9.857

g. $\sqrt{23+11} = \sqrt{34}$
$\approx 5.83095\ldots$
≈ 5.831

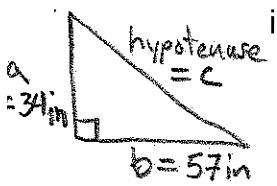
h. $\sqrt{\frac{15}{31}} \approx 0.695608\ldots$
≈ 0.696

5 DWK

2,6457
+7,2111
9,8568

Solve the following word problem, showing all steps. You must identify the quantity that the variable stands for, write an equation, solve the equation and then write out your answer in English words.

i. Find the hypotenuse of a right triangle that has legs of 34 inches and 57 inches. Give an exact answer, and then use a calculator to round your answer to three decimal places.



$c = \sqrt{a^2+b^2}$
$c = \sqrt{(34\text{ in})^2 + (57\text{ in})^2}$
$c = \sqrt{1,156\text{ in}^2 + 3,249\text{ in}^2}$
$c = \sqrt{4,405\text{ in}^2}$
$c = \sqrt{4,405}$ in
$c \approx 66.3701\ldots$ in
$c \approx 66.370$ in , or $c \approx 66.37$ in

ANS: The hypotenuse is 66.37 inches.

5 DWK

1 1
34
×34
136
+1020
1,156

3
57
×57
399
+2850
3,249

1 1
3,249
+1,156
4,405

Answers to Practice Problems:
a. 8; b. 7; c. 10; d. 5/7; e. 11.180; f. 9.857; g. 5.831
h. 0.696; i. The hypotenuse is 66.370 inches long.

Note: Portions of this document are excerpted from the textbook *Prealgebra*, 7th ed. by Charles McKeague

Section 5.8 Simplifying Square Roots

1. Multiplication Property for Square Roots: If a and b are positive numbers, then

$$\sqrt{a \bullet b} = \sqrt{a} \bullet \sqrt{b}$$

In words, the square root of a product is the product of the square roots.

Example 1: Simplify.

a. $\sqrt{49x} = \sqrt{49} \bullet \sqrt{x} = 7\sqrt{x}$

b. $\sqrt{9x} = \sqrt{9} \cdot \sqrt{x}$, since $3^2 = 9$

$= 3\sqrt{x}$

2. Repeated Factor Property for Square Roots: If a is a positive number, then

$$\sqrt{a \bullet a} = a \text{ OR } \sqrt{a^2} = a$$

Example 2: Simplify. Assume all variables represent positive numbers.

a. $\sqrt{6 \bullet 6} = 6$

b. $\sqrt{x \bullet x} = x$

c. $\sqrt{y^2} = y$

d. $\sqrt{9a^2} = \sqrt{9} \cdot \sqrt{a^2}$ or $\sqrt{9a^2} = \sqrt{(3a)^2}$

$= 3 \cdot a$ $= 3a$

$= 3a$

3. Simplifying Square Roots: When the expression under the square root has been completely factored, any factor that occurs twice can be taken out from under the square root symbol. Note: The factor occurs **twice** under the square root but **once** when brought outside the square root.

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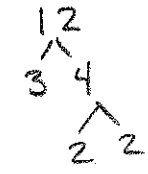
SDWK

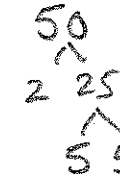
$12 = 2 \cdot 2 \cdot 3$
$12 = 4 \cdot 3$

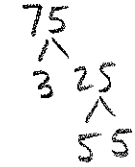
$50 = 2 \cdot 5 \cdot 5$
$50 = 25 \cdot 2$

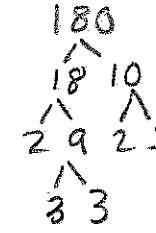
$75 = 3 \cdot 5 \cdot 5$
$75 = 25 \cdot 3$

$180 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$
$180 = 4 \cdot 9 \cdot 5$









Example 3: Simplify.

a. $\sqrt{12} = \sqrt{4} \cdot \sqrt{3}$
$= 2\sqrt{3}$

b. $\sqrt{50} = \sqrt{25} \cdot \sqrt{2}$
$= 5\sqrt{2}$

c. $\sqrt{75x^2} = \sqrt{25}\sqrt{x^2}\sqrt{3}$
$= 5 \cdot x \cdot \sqrt{3}$
$= 5x\sqrt{3}$

d. $\sqrt{180x^3} = \sqrt{4}\sqrt{9} \cdot \sqrt{x^2}\sqrt{5x}$
$= 2 \cdot 3 \cdot x \cdot \sqrt{5x}$
$= 6x\sqrt{5x}$

Practice Problems

Simplify each of the following without using a calculator.

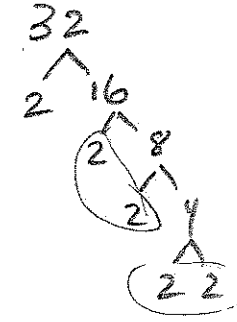
SDWK

$81 = 9^2$

$25 = 5^2$

$32 = 4 \cdot 4 \cdot 2$
$32 = 16 \cdot 2$

$8 = 2 \cdot 2 \cdot 2$
$8 = 4 \cdot 2$



a. $\sqrt{81x} = \sqrt{81}\sqrt{x}$
$= 9\sqrt{x}$

b. $\sqrt{a \bullet a} = a$

c. $\sqrt{25z^2} = \sqrt{25} \cdot \sqrt{z^2}$
$= 5 \cdot z$
$= 5z$

d. $\sqrt{32x} = \sqrt{16}\sqrt{2x}$
$= 4\sqrt{2x}$

e. $\sqrt{8y^3} = \sqrt{4} \cdot \sqrt{y^2} \cdot \sqrt{2y}$
$= 2 \cdot y \cdot \sqrt{2y}$
$= 2y\sqrt{2y}$

Answers to Practice Problems:

a. $9\sqrt{x}$; b. a; c. $5z$; d. $4\sqrt{2x}$; e. $2y\sqrt{2y}$

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Section 5.9 Adding and Subtracting Square Roots

1. Combining Similar Square Roots: We add or subtract square roots in the same way that we add similar terms. Two square roots can be added or subtracted if the expressions under the square root are **identical.** The addition or subtraction is performed by using the distributive property.

Example 1: Simplify. Give exact answers.

a. $4\sqrt{2}+3\sqrt{2}=4\bullet\sqrt{2}+3\bullet\sqrt{2}=(4+3)\sqrt{2}=7\sqrt{2}$

b. $5\sqrt{3}+7\sqrt{3}$ $=(5+7)\sqrt{3}$
$=12\sqrt{3}$

c. $7\sqrt{5}-11\sqrt{5}$ $=(7-11)\sqrt{5}$
$=-4\sqrt{5}$

d. $21\sqrt{11}-\sqrt{11}$ $=(21-1)\sqrt{11}$
$=20\sqrt{11}$

e. $\sqrt{13}+\sqrt{11}$ $=\sqrt{13}+\sqrt{11}$
(different)

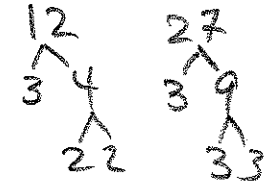
2. Adding and Subtracting Square Roots When Simplification is Required First: If the expressions under the square root are not identical, then the square roots can't be added or subtracted. However, sometimes the square roots can be simplified and then added or subtracted.

Example 2: Simplify. Give exact answers.

a. $\sqrt{12}+\sqrt{27}$

$=\sqrt{4}\cdot\sqrt{3}+\sqrt{9}\cdot\sqrt{3}$
$=2\sqrt{3}+3\sqrt{3}$
$=5\sqrt{3}$

SOLVE
$12=4\cdot3$
$27=9\cdot3$



Note: Portions of this document are excerpted from the textbook *Prealgebra,* 7th ed. by Charles McKeague

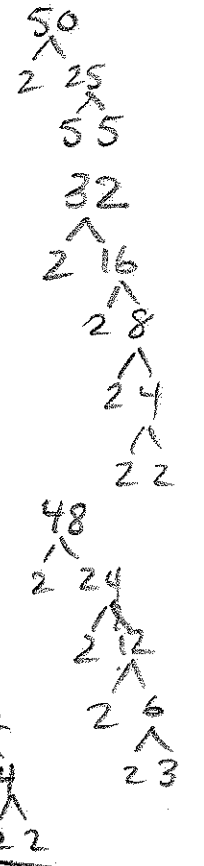
SDWK

$50 = 2 \cdot 5 \cdot 5$
$50 = 25 \cdot 2$

$32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
$32 = 4 \cdot 4 \cdot 2$
$32 = 16 \cdot 2$

$48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$
$48 = 9 \cdot 4 \cdot 3$
$48 = 16 \cdot 3$

$12 = 2 \cdot 2 \cdot 3$
$12 = 4 \cdot 3$



b. $\sqrt{50x} - \sqrt{32x} = \sqrt{25}\cdot\sqrt{2x} - \sqrt{16}\cdot\sqrt{2x}$
$= 5\sqrt{2x} - 4\sqrt{2x}$
$= \sqrt{2x}$

c. $8\sqrt{48} + 2\sqrt{12} = 8\cdot\sqrt{16}\sqrt{3} + 2\cdot\sqrt{4}\cdot\sqrt{3}$
$= 8 \cdot 4 \cdot \sqrt{3} + 2 \cdot 2 \cdot \sqrt{3}$
$= 32\sqrt{3} + 4\sqrt{3}$
$= 36\sqrt{3}$

3. Using Your Calculator to Find an Approximation: If an expression contains square roots that can't be added or subtracted because they aren't similar, you can use your calculator to find an approximation for the quantity. Round your answer to the decimal place indicated in the directions.

Example 3: Use your calculator to find an approximation for each expression. Round your answer to the nearest thousandth (three decimal places).

a. $\sqrt{21} \approx 4.58255\ldots$
≈ 4.583

b. $\sqrt{13} + \sqrt{11} \approx 3.605551\ldots + 3.316624\ldots$
$\approx 3.6056 + 3.3166$
≈ 6.9222
≈ 6.922

c. $7\sqrt{5}$
$\approx 7 \cdot (2.23606797\ldots)$
$\approx 7 \cdot (2.236067)$
≈ 15.652249
≈ 15.652

d. $\sqrt{13} - 5\sqrt{11}$
$\approx 3.605551\ldots - 5(3.316624\ldots)$
$\approx 3.6056 - 5(3.31662)$
$\approx 3.6056 - 16.5831$
≈ -12.9775
≈ -12.978

SDWK

$3.6056 + 3.3166 = 6.9222$

2.236067×7 5 places $= 15.652249$

3.31662×5 5 places $= 16.5831$

$16.5831 - 3.6056 = 12.9775$

Practice Problems:

Simplify each of the following. Give an exact answer.

a. $9\sqrt{3} - 5\sqrt{3}$ $= (9-5)\sqrt{3}$
$= 4\sqrt{3}$

b. $4\sqrt{7} - 6\sqrt{7}$ $= (4-6)\sqrt{7}$
$= -2\sqrt{7}$

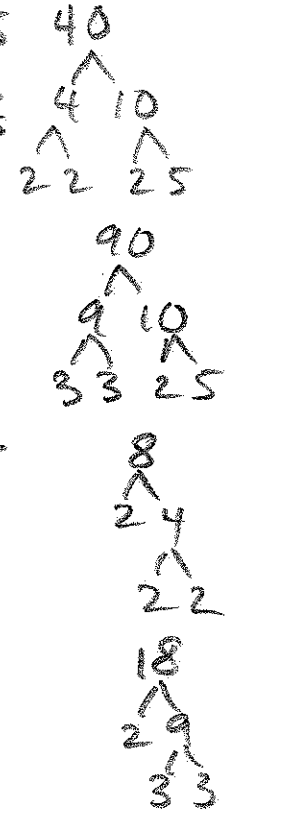
c. $\sqrt{19} + \sqrt{10}$ $= \sqrt{19} + \sqrt{10}$
(Different)

d. $5\sqrt{40} - 2\sqrt{90} + 3\sqrt{10}$
$= 5\cdot\sqrt{4}\sqrt{10} - 2\cdot\sqrt{9}\sqrt{10} + 3\sqrt{10}$
$= 5\cdot 2\cdot\sqrt{10} - 2\cdot 3\cdot\sqrt{10} + 3\sqrt{10}$
$= 10\sqrt{10} - 6\sqrt{10} + 3\sqrt{10}$
$= 7\sqrt{10}$

e. $\sqrt{8x} - \sqrt{18x}$
$= \sqrt{4}\sqrt{2x} - \sqrt{9}\sqrt{2x}$
$= 2\sqrt{2x} - 3\sqrt{2x}$
$= -\sqrt{2x}$

SPWK

$40 = 2\cdot 2\cdot 2\cdot 5$
$40 = 4\cdot 10$
$90 = 3\cdot 3\cdot 2\cdot 5$
$90 = 9\cdot 10$
$8 = 2\cdot 2\cdot 2$
$8 = 4\cdot 2$
$18 = 2\cdot 3\cdot 3$
$18 = 9\cdot 2$



Use a calculator to find an approximation for the given expression.
Round your answer to three decimal places.

f. $2\sqrt{57} - \sqrt{88}$

$\approx 2\cdot(7.549834\ldots) - 9.380831$
$\approx 2\cdot(7.54983) - 9.38083$
$\approx 15.09966 - 9.38083$
≈ 5.71883
≈ 5.719

SPWK

7.54983 5 places
× 2
15.09966

15.09966
−9.38083
5.71883

Answers to Practice Problems:

a. $4\sqrt{3}$; b. $-2\sqrt{7}$; c. Can't add these unless you approximate with a calculator; d. $7\sqrt{10}$; e. $-\sqrt{2x}$; f. 5.719

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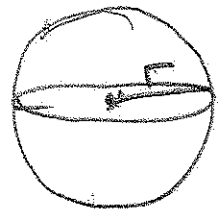

$r = \frac{1}{2}d$

$r = \frac{1}{2}(23.45\text{ yds})$

$r = 11.725\text{ yds}$

$d = 23.45\text{ yds}$

SCRATCH

$$\begin{array}{r} 11.725 \\ 2\overline{)23.450} \\ -2 \\ \hline 3 \\ -2 \\ \hline 14 \\ -14 \\ \hline 5 \\ -4 \\ \hline 10 \\ -10 \\ \hline 0 \end{array}$$

★★ Round to nearest Hundredth

f. Find the volume of a sphere of diameter 23.45 yds.
(Hint: First find the radius of the sphere.)

$V = \frac{4}{3}\pi r^3$, $r = 11.725\text{ yds}$

$V = \frac{4}{3}(3.14)(11.725\text{ yds})^3$

$V \approx \frac{4}{3}(3.14)(1{,}611.901703)\text{ yd}^3$

$V \approx \frac{4}{3}(5{,}061.371347)\text{ yd}^3$

$V \approx 20{,}245.485388\text{ yd}^3$

$V \approx 6{,}748.495129\text{ yd}^3$

$V \approx 6{,}748.50\text{ yd}^3$

or

$V \approx 6{,}748.5\text{ yd}^3$

Answers to Practice Problems: a. $0.2\overline{7}$; b. 0.29; c. 0.2; d. 0.8; e. 1.54; f. 6748.50 yds^3

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